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MAGNETOACOUSTIC SHOCK WAVES IN A NONUNIFORM PLASMA FLOW

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The flow of the solar wind around the planets and other bodies generates a weak magnetoacoustic shock wave [1]. The problem of describing a shock wave in a nonuniform plasma flow arises in this and similar situations [2]. Here we propose an approximate method of describing the field in the vicinity of a magnetoacoustic shock front. Based on the geometrical-acoustics (ray) description, this field is represented by a series, in which second-order small terms are taken into account by solving a Riccati equation. The magnetoacoustic shock intensity is estimated, and a relation is derived between the velocity of the shock front and the cross section of the ray tube. An algorithm is proposed for converting the fields from the moving frame to the laboratory frame.

1. We describe the field in the cold plasma by the magnetohydrodynamic (MHD) equations

$$\begin{gathered}\partial\rho/\partial t + \operatorname{div}(\rho\mathbf{v}) = 0,\ \operatorname{div}\mathbf{H} = 0,\\ \partial\mathbf{H}/\partial t - \operatorname{curl}[\mathbf{v}, \mathbf{H}] = 0,\ d\mathbf{v}/dt - (\mu/\rho)[\operatorname{curl}\mathbf{H}, \mathbf{H}] + \mathbf{g}(r) = 0.\end{gathered} \tag{1.1}$$

Here ρ, $\mathbf{v}$, and $\mathbf{H}$ are the density of the plasma, the velocity, and the magnetic field, and μ and g are the permeability and the gravitational acceleration. The subscript 0 refers to the unperturbed field: $\rho_0 = \rho_0(z)$, $\mathbf{v}_0 = v_0(z)\mathbf{e}_x$, $\mathbf{H}_0 = \text{const}$. The fields are perturbed by the presence of a fixed smooth body, around which the plasma moves in a nonuniform flow. We transform to a local coordinate system (frame) associated with the flow velocity $v_0(z)\mathbf{e}_x$ at

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the height of the observation point. It differs from the frame introduced in [3], where the frame is associated with the flow velocity at the height of the perturbing body. Consequently, in our frame (in which the fields and coordinates are primed) the plasma is at rest in the unperturbed state: $\mathbf{v}_0{}' = 0$. In the cold stationary plasma the magnetoacoustic wave has the following property in the linear approximation: Energy transport is in the same direction as the perpendicular to the wave front. We therefore have "isotropicity" in the above-mentioned restricted sense. This consideration facilitates the description of the wave process in orthogonal ray coordinates α', β', δ'. The vector α' is perpendicular to the wave front; β' and δ' lie on the surface of the wave front.

Here we consider the situation in which the Čerenkov radiation mechanism is in effect [$v_0 > b_0 \equiv (\mu H_0{}^2/\rho_0)^{1/2}$, where b_0 is the Alfvén velocity] and a weak shock wave has been generated. To simplify the description of the fields, it is convenient to orient the β' axis normal to $\mathbf{H}_0'$: $\beta' \perp \mathbf{H}_0'$. This is equivalent to directing β' along the line of intersection of the shock front with the plane perpendicular to $\mathbf{H}_0{}'$. The curving of the shock front in the course of its propagation causes β' to rotate about α'. For a magnetoacoustic wave v' does not have a projection onto the δ' axis in the frame α', β', δ' The curvilinear orthogonal coordinates α', β', δ' are related to the laboratory coordinates by the equation

$$(h_1'd\alpha')^2 + (h_2'd\beta')^2 + (h_3'd\delta')^2 = dx^2 + dy^2 + dz^2$$

[$h_i'(\alpha', \beta', \delta')$ are the Lamé constants]. The wave front (the shock front in particular) is described by the expression

$$t - T'(\alpha', \beta_1', \delta_1') = 0, \ \ \beta_1' = \text{const}, \ \ \delta_1' = \text{const} \tag{1.2}$$

(β_1' and δ_1' characterize the point of intersection of the coordinate line with the front). The direction of α' is given by the unit vector

$$\mathbf{e}_{\alpha'} = (\nabla)'T'/|(\nabla)'T'|, \tag{1.3}$$

and the velocity of the front u' is written in the form

$$u'(\alpha', \beta', \delta') = (d\mathbf{r}'/dt, \ \mathbf{e}_{\alpha'}). \tag{1.4}$$

As a result,

$$h_1' = u'(\alpha', \beta', \delta')\left[u'(\alpha_1', \beta_1', \delta_1')\right]^{-1}. \tag{1.5}$$

For the function T' to depend only on the longitudinal coordinate α', the Lamé constant h_1' must be chosen so that $h_1' \sim u'$. We assume below that $h_1' = u'(\alpha', \beta', \delta')[u'(\alpha_1', \beta_1', \delta_1')]^{-1}$, where $\alpha = \alpha_1$ characterizes the initial value of α'. Equation (1.5) is an eikonal equation. In the frame α', β', δ' we have the relation

$$(\mathrm{div})'\left(\mathbf{e}_{\alpha'}/h_2'h_3'\right) = 0, \tag{1.6}$$

which represents a transport equation. We set $h_2'\, h_3' = A'$ (A' is the dimensionless cross section of a narrow ray tube). This choice of the constants h_2' and h_3' enables us to make the limiting transition to the approximation of linear theory.

We investigate the situation in which the perturbation scale in the directions β' and δ' is smaller than the radius of curvature of the front and the inhomogeneity scale of the plasma in the investigated region. This condition permits us to regard the perturbations as locally homogeneous and to ignore the derivatives in the directions β' and δ' perpendicular to the ray tube. We then obtain the following approximation for a magnetoacoustic wave:

$$\begin{gathered}(\mathrm{div})'\mathbf{v}' \approx \frac{1}{A'}\frac{1}{h_1'}\frac{\partial}{\partial\alpha'}\left(A'v_{\alpha'}'\right), \ (\mathrm{curl})\mathbf{H}' \approx \frac{1}{A'}\frac{1}{h_1'}\frac{\partial}{\partial\alpha'}\left(A'H_{\beta'}'\right)\mathbf{e}_{\delta'}, \\ (\mathrm{curl}\,)'\left[\mathbf{v}', \mathbf{H}'\right] \approx \frac{1}{h_1'}\frac{\partial}{\partial\alpha'}\left(H_{\alpha'}'v_{\beta'}' - H_{\beta'}'v_{\alpha'}'\right)\mathbf{e}_{\beta'}.\end{gathered} \tag{1.7}$$

In this problem the perturbation of the fields $(\rho' - \rho_0')$, $(\mathbf{H}' - \mathbf{H}_0')$, $\mathbf{v}'$, $(A' - A_0')$ (we assume below that $A_0' = 0$) is attributable to the presence of a body moving through the plasma, so that the absence of the body corresponds to the absence of any perturbation of the field. For the nonrelativistic flow velocities discussed here, $\mathbf{H}_0' = \mathbf{H}_0 = \text{const}$. Taking Eqs. (1.7) into account, we write Eqs. (1.1) for a magnetoacoustic wave far from the body in primed coordinates:

$$\frac{\partial \rho'}{\partial t} + \frac{1}{A' h_1'} \frac{\partial}{\partial \alpha'} \left(A' \rho' v'_{\alpha'}\right) \approx G_1; \tag{1.8}$$

$$\frac{1}{h_1'} \frac{\partial}{\partial \alpha'} \left(A' H'_{\alpha'}\right) \approx 0; \tag{1.9}$$

$$\frac{\partial H'_{\beta'}}{\partial t} - \frac{1}{h_1'} \frac{\partial}{\partial \alpha'} \left(H'_{\alpha'} v'_{\beta'} - H'_{\beta'} v'_{\alpha'}\right) \approx G_2; \tag{1.10}$$

$$\frac{\partial v'_{\alpha'}}{\partial t} + \frac{v'_{\alpha'}}{h_1'} \frac{\partial v'_{\alpha'}}{\partial \alpha'} + \frac{\mu H'_{\beta'}}{\rho' A' h_1'} \frac{\partial}{\partial \alpha'} \left(A' H'_{\beta'}\right) + g'_{\alpha'} \approx G_3; \tag{1.11}$$

$$\frac{\partial v'_{\beta'}}{\partial t} + \frac{v'_{\alpha'}}{h_1'} \frac{\partial v'_{\beta'}}{\partial \alpha'} - \frac{\mu H'_{\alpha'}}{\rho' A' h_1'} \frac{\partial}{\partial \alpha'} \left(A' H'_{\beta'}\right) + g'_{\beta'} \approx G_4, \tag{1.12}$$

where $G_1 = v_0 \partial \rho'/\partial x'$; $G_2 = v_0 \partial H'_{\beta'}/\partial x'$; $G_3 = v_0 \partial v'_{\alpha'}/\partial x'$; $G_4 = v_0 \partial v'_{\beta'}/\partial x'$; $x' = x - v_0 t$. We base the ensuing discussion on the satisfaction of a condition that permits the influence of the functions G_i in Eqs. (1.8)-(1.12) to be disregarded:

$$\left|\frac{1}{h_1'} \frac{\partial f'}{\partial \alpha'}\right| \gg \left|\frac{v_0}{v'_{\alpha'}} \frac{\partial f'}{\partial x'}\right| = \left|\frac{v_0}{v'_{\alpha'}} \cos \Omega'\right| \left|\frac{1}{h_1'} \frac{\partial f'}{\partial \alpha'}\right|$$

(Ω' is the angle between the vectors $\boldsymbol{\alpha}'$ and $\mathbf{x}'$, and f' stands for any one of the functions ρ', $v'_{\alpha'}$, $v'_{\beta'}$, $H'_{\beta'}$). As a result, we have a constraint on the orientation of the axis of the ray tube and on the freestream velocity v_0:

$$b_0 < |v_0| \ll \left|v'_{\alpha'} (\cos \Omega)^{-1}\right|.$$

The unperturbed state of the fields corresponds to the case $\partial v_{0\alpha'}/\partial \alpha' = 0$, $\partial v_{0\beta'}/\partial \beta' = 0$, $\partial A_0'/\partial \alpha' = 0$, $g' = 0$, $A_0' = 0$. One way in which the ray tubes for magnetoacoustic waves differ from the ray tubes for a neutral is the two-dimensional character of the plasma motion: The velocity has a transverse component $v'_{\beta'} \neq 0$. Consequently, a mass flux sets in through the lateral surface of the tube. However, the energy flux through the lateral surface is zero; the tubes are isolated from one another in the energy sense.

We investigate the fields only in the vicinity of the front (1.2). Two small parameters are involved in the solution of the problem: weak nonlinearity and the small distance from the shock front (1.2), so that one-dimensional field in narrow ray tubes can be described in series form:

$$F'(t, \alpha') = \sum_{n=0}^{\infty} F'_n(\alpha') (t - T')^n, \quad A'(\alpha') = \sum_{n=0}^{\infty} A'_n(\alpha').$$

Here $\partial F'/\partial t' = F_1' + 2F_2'(t - T') + \ldots$, $\partial F'/\partial \alpha' = dF_0/d\alpha' - F_2' dT'/d\alpha' + (t - T')(dF_2'/d\alpha' - 2F_2' dT'/d\alpha') + \ldots$. A similar series expansion has been used [3] to describe a sonic boom in a nonuniform atmospheric flow. Restricting the series to second-order terms, from the system (1.8), (1.9) we obtain the following condition for its solvability in the approximation $G_i \approx 0$:

$$\frac{b_0'(\alpha')}{h_1'} \frac{dT'}{d\alpha'} = \pm 1, \quad h_1' = \pm b_0'(\alpha') \left[b_0'(\alpha_1')\right]^{-1}. \tag{1.13}$$

The eikonal equation (1.13) corresponds to the equation for the characteristics C_+ and C_- in the case of the linear approximation. Since we are interested in the outgoing wave, we choose the plus sign in Eq. (1.13) and obtain a Riccati equation for $v'_{1\alpha'}$ from Eqs. (1.8)-(1.12) with $G_i = 0$:

$$\frac{dv'_{1\alpha'}}{d\alpha'} - \frac{3}{2} \left(\frac{v'_{1\alpha'} \sqrt{h_1'}}{b_0'}\right)^2 + D' v'_{1\alpha'} = 0, \tag{1.14}$$

where

$$D' = \frac{1}{2}\frac{d}{d\alpha'}\ln\frac{A_1'}{b_0' H_{0\beta'}'}. \tag{1.15}$$

The fact that $v_0' = 0$ and $dv_0'/d\alpha' = 0$ simplifies the expression for D' considerably. This is one of the advantages of choosing a frame attached to the flow (see [3] for comparison). In the linear approximation the solution of Eq. (1.14) has the form

$$v_{1\alpha'}' \approx v_{1\alpha'}'(\alpha_1')[B'(\alpha')]^{-1} \quad \left(B'(\alpha') = \exp\left(\int_{\alpha_1'}^{\alpha'} D'\,d\alpha'\right)\right).$$

A rigorous solution of Eq. (1.14) is given by the relation

$$v_{1\alpha'}' = [B'(\alpha')/v_{1\alpha'}'(\alpha_1') - \Phi']^{-1}, \quad \Phi'(\alpha', \alpha_1') = B'\int_{\alpha_1}^{\alpha}\frac{3h_1'}{2}(b_0')^{-2}(B')^{-1}d\alpha'.$$

For $\alpha' \gg \alpha_1'$, $\Phi' \gg B'[v_1{}'_{\alpha'}(\alpha_1')]^{-1}$ the function Φ' plays the role of a large parameter of the problem, and we can adopt the approximation

$$v_{1\alpha'}' \approx -[\Phi'(\alpha_1', 0)]^{-1}. \tag{1.16}$$

Note that the use of Eq. (1.16) prohibits the limiting transition to the linear approximation of the problem. The specific form of D' given by Eq. (1.15) can be used to write the function B' in the form

$$B' = [(A_1'(\alpha')/A_1'(0))(b_0'(0)/b_0'(\alpha'))]^{1/2} H_{0\beta'}'(0)/H_{0\beta'}'(\alpha').$$

It is evident from the basic Eqs. (1.8)-(1.12) that in order to determine the fields, it is sufficient to know the dimensionless function A' to within a constant factor. We shall use the normalization $A_1'(0) = 1$ below. The first-approximation functions are expressed in terms of $v_1{}'_{\alpha'}$ and for $G_i \approx 0$, according to Eqs. (1.8)-(1.12), have the form

$$v_{1\beta'}'/v_{1\alpha'}' = -H_{0\alpha'}'/H_{0\beta'}', \quad \rho_1'/\rho_0' = v_{1\alpha'}'/b_0',$$
$$H_{1\beta'}'/H_{0\beta'}' = v_{1\alpha'}'/b_0'(H_0/H_{0\beta'}')^2, \quad H_{1\alpha'}' = 0.$$

One possible approach to the refinement of Eq. (1.13) is to invoke the equation for the characteristics C_+ and C_-:

$$\frac{1}{h_1'}\frac{dt}{d\alpha'} = (v_{1\alpha'}' \pm b')^{-1},$$
$$h_1' = [v_{1\alpha'}'(\alpha') \pm b'(\alpha')][v_{1\alpha'}'(\alpha_1') \pm b'(\alpha_1')]^{-1}, \tag{1.17}$$

which corresponds to Eq. (1.15) with $u' = v_1{}'_{\alpha'} \pm b'$.

2. We now estimate the magnetoacoustic shock intensity I' and derive a relation between the velocity of the front u' and the cross section A':

$$I' \equiv (p' - p_0')/p_0' \approx p_1'/p_0'(t - T').$$

We accomplish this on the basis of the equation of state of a polytropic gas $p' = p_0'(\rho'/\rho_0')^\gamma$, γ = const, which is a simple way to model the function $p'(\rho_0')$. We write the intensity in the representation

$$I' \approx \gamma\rho_1'/\rho_0'(t - T') \approx -\gamma(t - T')[b_0'\Phi'(\alpha', 0)]^{-1}, \tag{2.1}$$

where it is required to determine (t − T'). The value $T_0' \equiv \alpha'/b_0'(\alpha_1')$ corresponds to the linear approximation of T' for the characteristic C_+. Introducing the new function $L'(\alpha') = T_0' - T'$, we describe the shock front by the relation

$$t - T_0' = -L'(\alpha'). \tag{2.2}$$

From Eq. (2.1) we obtain the following generalization of the eikonal equation (1.13) [another form of Eq. (1.17)]:

$$\frac{1}{h_1'}\frac{dt}{d\alpha'} = \frac{1}{b_0'(\alpha_1')} - \frac{dL'}{d\alpha'}. \tag{2.3}$$

The velocity of a low-intensity shock front is the arithmetic mean of the characteristic velocity before and after the front:

$$\begin{gathered} u' = h_1' d\alpha'/dT' \approx (1/2)\left[b_0'(\alpha_1') + (v_{1\alpha'}' + b')\right] \approx b_0'(\alpha_1') + \\ + (1/2)\left(v_{1\alpha'}' + b_1'\right)(t - T) \approx b_0'(\alpha_1') - (L'/2)\left(v_{1\alpha'}' + b_1'\right). \end{gathered} \tag{2.4}$$

Equations (2.3) and (2.4) can be used to find a differential equation for the correction L':

$$(2/L')\,dL'/d\alpha' = -\left(v_{1\alpha'}' + b_1'\right)\left(b_0'(\alpha_1')\right)^{-2}, \quad L' = K_0\,[\Phi'(\alpha', 0)/B']^{1/2}.$$

The parameter K_0 = const takes into account the shape of the body and the boundary condition on its surface. It is impossible to determine K_0 within the framework of the given method. A representation for K_0 can be obtained by other methods (see, e.g., [4]) in the solution of the simpler problem v_0 = const, b_0 = const. We write Eq. (2.1) in the form

$$I' = (2/3)\,K_0\left[b_0'(\alpha_1')\right]^{-1}\gamma\,(B'\Phi')^{-1/2}.$$

In the special case of a homogeneous medium and for v_0 = const we have $B' \sim (A')^{1/2}$ and

$$I' \sim \left[A' \int_{\alpha_1'}^{\alpha'} (A')^{-1/2}\,d\alpha'\right]^{-1/2}.$$

For a conical shock front in this situation, $A' \sim r'$ (r' is the radial distance in cylindrical coordinates), and $I' \sim (r')^{-3/4}$ is Landau's well-known asymptotic representation for axisymmetrical shock fronts. Equation (2.4) can be used to find the most important relation for the subsequent solution of the problem, namely the relation between the velocity of the shock front and the cross section of the ray tube:

$$\begin{gathered} u'(A') = F'(A') \\ \left(F'(A') \approx b_0'(\alpha_1') + \frac{3}{4}K_0\,(B'\Phi')^{-1/2}\right). \end{gathered} \tag{2.5}$$

3. The existence of the relation (2.5) between u' and A' permits the well-known "shock dynamics" approximation to be used in the primed coordinate system, provided that the properties of the flow vary sufficiently slowly in the unperturbed state. Such a description is essentially a nonlinear generalization of the approximation of linear ray theory. We use Eqs. (1.5) and (1.6) for this purpose. Of fundamental importance is the fact that $e_{\alpha'}$ [the unit vector in Eq. (1.3)] coincides with the ray direction (the direction of energy transport) and is perpendicular to the shock front. The function $e_{\alpha'}$ characterizes the eikonal, and Eq. (1.5) generalizes the representations (1.13) and (2.4). Oddly enough, Eqs. (1.5) and (1.6) are suitable for the description of both weak and strong shock waves [5]. We have therefore obtained a closed system of equations (1.3), (1.5), (1.6), (2.5). The approximation (1.3), (1.5), (1.6) can be used by virtue of the local immobility of the medium in the primed coordinate system and the orthogonality of the rays to the shock front. In the primary frame the eikonal equation and the transport equation acquire the form

$$|\nabla T| = u^{-1}; \tag{3.1}$$

$$\operatorname{div}\,(\mathbf{e}_\alpha/A) = 0. \tag{3.2}$$

Here, in contrast with Eq. (1.3), the ray direction e_α is not orthogonal to the shock front, because isotropicity does not exist in the above-mentioned sense in the primary frame. For finding e_α we proceed as follows. We write Eq. (1.5) in the Hamilton–Jacobi form $\mathscr{H}' = 0$, where $\mathscr{H}'$ is the Hamiltonian:

$$\mathscr{H}'\left(\frac{\partial T'}{\partial x'}, \frac{\partial T'}{\partial y'}, \frac{\partial T'}{\partial z'}; T'\right) = \frac{1}{2}\left[\left(\frac{\partial T'}{\partial x'}\right)^2 + \left(\frac{\partial T'}{\partial y'}\right)^2 + \left(\frac{\partial T'}{\partial z'}\right)^2 - \left(\frac{1}{u'(T')}\right)^2\right]. \tag{3.3}$$

Note that $e_{\alpha'}$ in Eq. (1.3) has the following significance with the Hamiltonian $\mathscr{H}'$:

$$\mathbf{e}_{\alpha'} = \frac{\partial \mathscr{H}'}{\partial ((\nabla)' T')} \left| \frac{\partial \mathscr{H}'}{\partial ((\nabla)' T')} \right|^{-1}.$$

The unit vector e_α in the ray direction is analogous:

$$\mathbf{e}_\alpha = \partial \mathscr{H}/\partial(\nabla T)|\partial \mathscr{H}/\partial(\nabla T)|^{-1} = l_1\mathbf{e}_x + l_2\mathbf{e}_y + l_3\mathbf{e}_z, \quad dx' = dx - v_0(z)dT,\ dy' = dy,\ dz' = dz. \tag{3.4}$$

All that remains is to find a representation for the Hamiltonian $\mathscr{H}$ in the primary frame. It follows from the relation dT = dT' that

$$\partial T'/\partial x_i' = (\partial T/\partial x_i)(1 - v_0 \partial T/\partial x)^{-1}, \quad i = 1, 2, 3,\ x_1 = x,\ x_2 = y,\ x_3 = z, \tag{3.5}$$

from Eqs. (3.3) and (3.5) we have

$$\mathscr{H} = \frac{1}{2}\left\{\left(1 - v_0\frac{\partial T}{\partial x}\right)^{-2}\left[\left(\frac{\partial T}{\partial x}\right)^2 + \left(\frac{\partial T}{\partial y}\right)^2 + \left(\frac{\partial T}{\partial z}\right)^2\right] - \frac{1}{u^2(T)}\right\},$$

and we obtain the direction cosines ℓ_1, ℓ_2, and ℓ_3 in Eq. (3.4) in the form

$$l_1 = \frac{1}{R_1}\left[\frac{\partial T}{\partial x} + v_0\left(\frac{\partial T}{\partial y}\right)^2 + v_0\left(\frac{\partial T}{\partial z}\right)^2\right],$$

$$l_2 = \frac{1}{R_1}\frac{\partial T}{\partial y}\left(1 - v_0\frac{\partial T}{\partial x}\right),$$

$$l_3 = \frac{1}{R_1}\left[\frac{\partial T}{\partial z}\left(1 - v_0\frac{\partial T}{\partial x}\right)\right], \quad R_1 = \left\{\left[\left(\frac{\partial T}{\partial x}\right)^2 + \left(\frac{\partial T}{\partial z}\right)^2\right]\left[1 + v_0^2\left(\frac{\partial T}{\partial z}\right)^2\right] + \left(\frac{\partial T}{\partial y}\right)^2\left[1 + v_0^2\left(\frac{\partial T}{\partial x}\right)^2 + v_0^2\left(\frac{\partial T}{\partial y}\right)^2 + 2v_0^2\left(\frac{\partial T}{\partial z}\right)^2\right]\right\}^{1/2}.$$

We now determine the relations between the cross sections A and A'. Since A is the cross section of the ray tube in the laboratory frame, A' is the area of the surface "cut out" by the tube on the shock front. Bearing this fact in mind, we have

$$A = A'(\mathbf{e}_\alpha, \mathbf{n}), \tag{3.6}$$

where $\mathbf{n}$ is the unit normal to the shock front:

$$\mathbf{n} = \nabla T/|\nabla T|. \tag{3.7}$$

Equation (2.5) can be used to obtain the required relation between u and A:

$$u(A) = F(A). \tag{3.8}$$

As a result, we have the closed system of equations (3.1), (3.2), (3.4), (3.6)-(3.8), which can be solved numerically on a computer. We note that the description (3.1), (3.2), (3.4), (3.6) generalizes the data of [6], in which the case of uniform flow in a homogeneous medium is analyzed, and a more cumbersome and less constructive method than (3.4) is used to find the expression for e_α. This is done by transforming Eq. (1.6) into a fixed system and reducing it to divergence form. It is then possible to find e_α. The Hamiltonian $\mathscr{H}$ simplifies the procedure, making it possible to find e_α in Eq. (3.4) as the result of differentiating $\mathscr{H}$. The system (3.1), (3.2), (3.4), (3.6) is suitable for the description of shock waves of any intensity, provided that it is augmented with the appropriate relation between u and A. For weak shock waves, on the other hand, the situation is simplified by the closeness of the shock velocity u to the linear Alfvén velocity b_0; also, we have $A = \sum_{n=1}^{\infty} A_n \approx A_1$.

On the basis of the relation (2.5), the eikonal equation (3.1) acquires the form $\mathscr{H}$

$$\left(\frac{\partial T}{\partial x}\right)^2 + \left(\frac{\partial T}{\partial y}\right)^2 + \left(\frac{\partial T}{\partial z}\right)^2 = \left(1 - v_0\frac{\partial T}{\partial x}\right)^2\left[b_0 + \frac{3}{4}K_0(B\Phi)^{-1/2}\right]^{-2}, \tag{3.9}$$

where $(3/4)K_0(B\Phi)^{-1/2} \ll b_0$ for weak shock waves, so that Eq. (3.9) can be solved by successive approximations. The zeroth approximation corresponds to $K_0 = 0$, which is the first approximation for the ray description. In this approximation the ray paths are independent of the field: For $K_0 = 0$, Eq. (3.9) splits off from the system (3.2), (3.6)-(3.8); T(x, y, z) and $e_\alpha(x, y, z)$ are determined from the given functions $v_0(z)$ and $b_0(z)$.

The above-proposed scheme is based on rejection of the terms G_i in the system (1.8)-(1.12). The solution can be subsequently refined either by a successive-approximation procedure or by obtaining a Riccati equation for $G_i \neq 0$.

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ELECTRIC FIELD BUILDUP IN PORE COLLAPSE

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The rapid deformation and fracture of solids gives rise to strong electric fields with a resultant emission of particles, x-rays, and radio-frequency radiation from the fracture surface. The field is generated by the production and separation of point defects and charged dislocations on the shock front [1] and at the tips of growing cracks [2, 3]. In this paper we examine the electric effects arising near cavities and pores which collapse in a shock wave.

Consider a porous dielectric medium. When the material undergoes impact compression, the highest deformation rates occur in the plastic zones localized around cavities and inhomogeneous inclusions [4]. These zones are production sites of point defects and electrically charged dislocations. The defect multiplication rate is proportional to the shear deformation rate $d\gamma/dt$. At low concentrations the recombination of point defects can be neglected [1]. In the case of two defect types with opposite charges, the equation of continuity (i = 1, 2) has the form

$$\frac{\partial n_i}{\partial t} + \operatorname{div} \mathbf{j}_i = M\left|\frac{d\gamma}{dt}\right|, \quad \mathbf{j}_i = n_i\mathbf{v} - \delta \operatorname{grad} n_i\nu_i + \frac{\sigma_i}{q_i}\mathbf{E}. \tag{1}$$

Here M is the multiplication constant; n_i and q_i are the point-defect concentration and charge, or the number of dislocations per unit area and the charge per unit dislocation length; the defect current density $\mathbf{j}_i$ has three components, involving the lattice velocity of motion $\mathbf{v}$, the defect displacement relative to the lattice (diffusion), and the drift in the field of strength $\mathbf{E}$; ν_i is the displacement frequency of a defect over one interatomic distance δ ($\delta\nu_i$ is the dislocation velocity); σ_i is the ionic conductivity.

For rapidly varying loads, $\mathbf{j}_i = n\mathbf{v}_i$ at first approximation. Leaving the defect type unspecified, we substitute this in (1). Assuming the material surrounding the pores is incompressible, i.e., div v = 0, we obtain $n_i \equiv n = n_0 + M\gamma$ in Lagrangian coordinates (n_0 is the initial defect concentration).

In the following approximation we seek small corrections $m_i \ll n$. Setting in (1) $n_i = n + m_i$, we get

$$\frac{\partial m_i}{\partial t} + \operatorname{div} m_i\mathbf{v} - \delta^2 \Delta n\nu_i + \frac{1}{q_i}\operatorname{div}\sigma_i\mathbf{E} = 0, \tag{2}$$

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